DETERMINING THE STEADY-STATE RESPONSES IN RF CIRCUITS USING GMRES, CGS, AND BICGSTAB SOLUTION IN sSPICE FOR LINUX

Madeline A. Kleiner, Mohammed N. Afsar, Fellow IEEE

Department of Electrical Engineering and Computer Science
Tufts University
Medford, MA.

ABSTRACT

sSPICE, an extension of SPICE3f5 for Linux, is under development to determine the steady-state responses for RF integrated circuits. The shooting-Newton algorithm for steady-state determination is incorporated with solution of the resulting iterates using the Krylov-subspace methods, GMRES, CGS, and BICGSTAB. The direct method shows much more computer efficiency than the regular transient analysis in SPICE. Only small improvements are seen using the explicit matrix instead of Gaussian elimination methods. Using the implicit form of GMRES resulted in improvements over the explicit GMRES and over the shooting-Newton Gaussian elimination method.

INTRODUCTION

Determining the steady-state response for integrated circuits that are lightly damped and have large time constants is computationally expensive. Simulating high Q circuits is also computationally expensive and these types of circuits occur regularly in communication systems. Four methods are commonly used to determine the steady-state response [1]: transient analysis, finite difference, shooting-Newton, and harmonic balance. In SPICE, the most popular analog circuit simulator, the steady-state is determined by allowing the time-domain transient analysis to run until all the transient signals have died off and all that remains is the periodic steady-state. With many integrated circuits this simulation takes a long time. The use of SPICE, therefore, is limited to integrated circuits having few components. The finite difference method and the shooting-Newton method both use the direct method [2]. The direct method uses the determination of the steady-state as a solution to a two-point boundary value problem. But using the standard matrix methods of solution, this method is still too expensive. Lastly, the harmonic balance method is used extensively and works well for weakly nonlinear circuits, but becomes computationally expensive for strongly nonlinear circuits. SPICE can simulate these strongly nonlinear circuits, but with poor computer efficiency.

Much work has explored the use of newer numerical methods to take advantage of matrix sparsity and the freedom from having to explicitly form the matrix needed for solution. GMRES, a robust non-stationary iterative method has been extensively reported as a speed up for steady-state determination in the time-domain [3,4]. Both GMRES and QMR have been used in the harmonic balance method to speed up steady-state solution on linear circuits [5]. Numerical methods not included in the non-stationary iterative methods have also been explored. Homotopy methods, for example, have been used to find the steady-state solution [6], but, unfortunately, they were found to be computationally expensive.

sSPICE as presented here solves some of these problems. It is an extension of SPICE 3f5, the analog circuit simulator developed at the University of California Berkeley. It uses the same direct methods for steady-state determination as incorporated into the SPICE 3c1-version [6,7]. GMRES, CGS, and BICGSTAB, all non-stationary iterative methods are incorporated into the code and compiled on Linux. A version of GMRES is available that uses the implicit matrix technique[4]. Users may choose either the standard transient analysis in SPICE or the shooting-Newton method in combination with the GMRES, CGS, or BICGSTAB method to solve the equations

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formed in the solution. Results from the comparison of the different methods are presented below.

1. ITERATIVE METHODS

Non-stationary iterative methods are an efficient means to solve non-symmetric linear systems, such as the large system of equations generated during the shooting-Newton method of steady-state determination. They are Krylov-subspace methods and variations on the Conjugate Gradient method for symmetric, positive definite systems. Iterative methods are used to solve the system represented as: Ax = b. The main differences between the non-symmetric iterative methods lies in (1) the method of constructing the basis vectors for the Krylov-subspaces and (2) the choice of the iterate [9]. For GMRES, an orthogonalization method, the matrix A itself is used. BICGSTAB and CGS use a biorthogonalization method. The matrix $\begin{bmatrix} A & 0 \\ 0 & A^T \end{bmatrix}$ is used to construct the Krylov subspace. The direct solvers such as Gaussian elimination have the workload of O(m) steps each requiring O(m^2) work for a total work estimate of O(m^3)[8]. This severely limits the number of equations that can make up the system being solved. The iterative methods theoretically require the same amount of work, but this applies only to the worst case. By using the sparsity and special structure of the large matrices generated by the shooting-Newton method of solution for the steady-state response, the amount of work can be reduced to O(m) at best or at least O(m^2). The Krylov-subspace algorithms also allow one to avoid the formation of the matrix A above. Instead of actually forming this matrix, its parts can be used in the matrix-multiplications needed for the calculations in the GMRES, CGS, and BICGSTAB. This method allows much larger circuits to be analyzed.

The sSPICE code implementing these methods was written in C and incorporated into the University of California Berkeley SPICE 3f5 [9]. Users can specify the direct steady-state determination using either the standard Gaussian elimination or the non-stationary iterative methods GMRES, CGS, or BICGSTAB.

2. STEADY-STATE

The determination of the steady-state response of many wireless communication devices driven by periodic inputs is of great interest to designers. If an analog circuit simulator such as SPICE is used for analysis, the differential equations generated by the system are integrated over time interval long enough for transient waveforms to die out. With rapidly decaying transients, SPICE can determine the steady-state response with great efficiency. But many of the RF circuits have transients that decay very slowly and it becomes almost impossible to integrate the differential equations over the entire transient response needed. Direct methods, such as the shooting-Newton method presented here, will find the initial state needed to put the circuit directly in steady-state [2]. If the circuit equations are represented as the system:

(1) $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t)$,

where **x** and **f** are n vectors, the vector **f** is periodic in time, t, and has a period of T. A constraint for achieving steady-state is that the transient effects have died off. This is represented by:

(2) $\mathbf{x}(0) = \mathbf{x}(T)$.

In other words, the solution at the end of the period is the same as the condition at the beginning of the period. This means that the circuit is in steady-state. The state transition function can be used to define the two-point boundary value problem, thus:

(3) $\mathbf{x}(0) - \phi(\mathbf{x}(t_0), t_0, T) = 0$,

where ϕ is the state transition function. The state transition function is implicitly derived; it is calculated at each timepoint until the end of the period. It is dependent on the initial state, x_0, the period of the response, T, and the starting time, t_0. Applying the shooting-Newton method to solve the boundary value problem results in the following iteration:

(4)

$$\mathbf{x}_0(t)^{k+1} = \mathbf{x}_0(t)^k - \left[\mathbf{I} - \mathbf{J}_\phi\right]^{-1}\left[\mathbf{x}_0{}^k - \phi(\mathbf{x}(t_0), t_0, T)\right]$$

where k is the iteration index and $\mathbf{J}_\phi$ is the sensitivity matrix and is represented by:

(5) $\mathbf{J}_\phi = \dfrac{d}{dx}\phi(\mathbf{x}(t_0), t_0, T)$.

The sensitivity matrix can be computed at the same time as the state transition function. Quantities needed for the calculation of the sensitivity matrix are already available at each timepoint from the transient analysis. One only uses these quantities and calculates the sensitivity matrix. The iterate is solved and, using a user defined limit, is considered converged or the circuit is simulated and another initial guess is used [2,7]. This process continues until the steady-state is reached. The shooting-Newton method computes a set of capacitor voltages and inductor currents for the circuit so that if these voltages and currents are used as the initial conditions for the transient analysis, the circuit is directly in steady-state.

The solution of this iterate equation is computationally expensive. Forming and solving the sensitivity matrix, because it is dense, is the most expensive. If a method like Gaussian elimination is used, then the costs are very high and the speed-up of the direct method of steady-state determination is limited. The newer iterative methods result in much greater computational savings. An even greater savings in computation time can be realized if instead of forming the sensitivity matrix at each transient analysis step, the quantities are stored and then accessed when needed.for the solution of the iterate in the Krylov-subspace method[1, 3].

Table 1: Test Circuits

Circuit #	Circuit	# States
1	DC Power Supply	4
2	Class C amplifier-low Q	5
3	Class C amplifier –high Q	5
4	Class C amplifier	11
5	Colpitts Oscillator- Q=50	3
6	Colpitts Oscillator-Q=100	3
7	Colpitts-Oscillator-high freq.	3
8	EC-Colpitts Oscillator	2

3. RESULTS

Table 1 lists the circuits used for simulation. Using SPICE3f5, their steady-state responses produce very long transient simulation times. The results are summarized in Figure 1 and Figure 2 for the shooting-Newton method of steady-state determination. The relative CPU time is the time of the iterative method divided by the time for Gaussian elimination. Using GMRES, CGS, and BICGSTAB iterative methods produce the expected significant improvement over regular transient analysis determination of steady-state. However, BICGSTAB seems to have difficulty converging to solution for some circuits, and for circuit 7 converged to the incorrect solution. Using the shooting-Newton algorithm for direct calculation of the steady-state response shows improvement over the transient analysis. However, large improvements expected from the iterative methods used to solve the shooting-Newton iterate are not evident. Using the implicit matrix method with GMRES resulted in more significant timesavings.

Because of memory overflow problems simulation of larger example circuits was not possible. The next round of examples will be run on a PC that has available more swap space available for Linux. Other non-stationary Krylov-subspace methods will be implemented and compared.

4. REFERENCES

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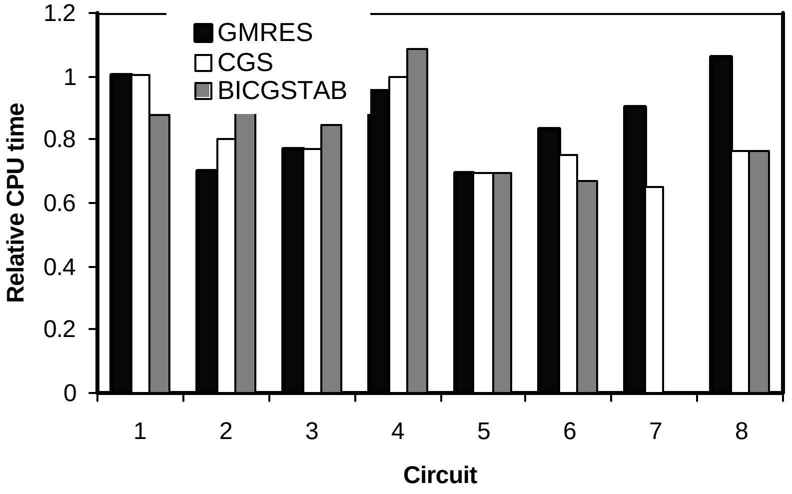

Figure 1. Comparison of GMRES, CGS, and BICGSTAB methods

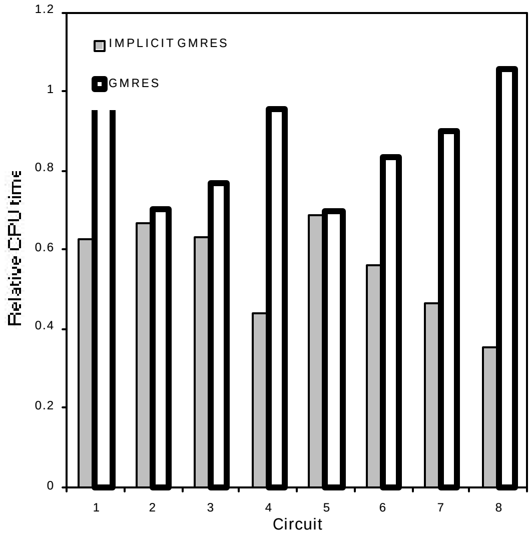

Figure 2. Comparison of explicit and implicit GMRES.

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