

## 2 LOSSLESS UNCOUPLED LINES

For the frequency range of operation under consideration, a quasi-TEM analysis of an IC interconnect is valid [3]. Under the quasi-TEM assumption for a uniform transmission line, the voltage  $V$  and the current  $I$  at any point  $x$  on a transmission line at time  $t$  is described by the telegrapher equations:

$$\frac{\partial V(x,t)}{\partial x} = -R I(x,t) - L \frac{\partial I(x,t)}{\partial t} \quad (1)$$

$$\frac{\partial I(x,t)}{\partial x} = -G V(x,t) - C \frac{\partial V(x,t)}{\partial t} \quad (2)$$

where  $R$ ,  $G$ ,  $L$  and  $C$  are, respectively, the per unit length series resistance, shunt conductance, inductance and capacitance of the transmission line.

Solving equations (1) and (2) as a function of time involves networks connected to the transmission line (called terminal networks). However, the solution to these equations consists of two travelling waves. One of these waves will be coming into a node and the other going out of the node. The voltage signals associated with these waves at a node "p" are denoted by  $V_p^{\text{in}}$  and  $V_p^{\text{out}}$  respectively. The

total signal at node p is the sum of these two signals.

For a lossless transmission line between nodes "a" and "b", the following relations hold:

$$V_b^{\text{in}}(t) = V_a^{\text{out}}(t - \delta) \quad (3)$$

$$V_a^{\text{in}}(t) = V_b^{\text{out}}(t - \delta) \quad (4)$$

where  $\delta$  is the propagation delay in the transmission line. These equations provide a convenient way to compute the incoming signal at a given time from the outgoing signal at a previous time. Consider a junction of more than one transmission lines (found in signal nets and more commonly found in power and clock nets). For example,  $m$  conductors meet at point p in Fig. 1. The outgoing signals at this point can be obtained from the incoming signals using the concept of scattering matrix [8].

The conductor with end nodes "k" and "p" is denoted by the index "k." The voltage wave on conductor k at point p travelling toward the node k is denoted by  $V_{pk}^{\text{out}}$  while the voltage wave incident at node p on conductor k is denoted by  $V_{pk}^{\text{in}}$ . Similar quantities can be defined on the other conductors. Then the relation among the signals coming into the node p and going out of the same node is given by

$$V_p^{\text{out}} = S V_p^{\text{in}} \quad (5)$$

where the vector  $V_p^{\text{out}}$  and  $V_p^{\text{in}}$  represent, respectively, the components  $V_{pk}^{\text{out}}$  and  $V_{pk}^{\text{in}}$  ( $k = 1, 2, \dots, m$ ) and  $S$  is a scattering matrix given by

$$S = \begin{bmatrix} S_{11} & S_{12} & \dots & S_{1m} \\ S_{21} & S_{22} & \dots & S_{2m} \\ \dots & \dots & \dots & \dots \\ S_{m1} & S_{m2} & \dots & S_{mm} \end{bmatrix} \quad (6)$$

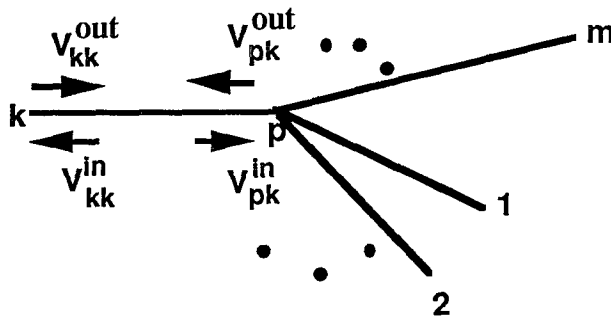


Fig. 1. A junction point p where  $m$  conductors meet.

$S_{jk}$  ( $j \neq k$ ) represents the fraction of  $V_{pk}^{\text{in}}$  transmitted to conductor  $j$  and  $S_{kk}$  represents the fraction of  $V_{pk}^{\text{in}}$  reflected back on conductor  $k$ . Notice that if  $V_{pj}^{\text{in}} = 0$  for all  $j \neq k$  (which is true if all conductors except conductor  $k$  are infinitely long or if the impedances are matched), then  $V_{pk}^{\text{out}} = S_{kk} V_{pk}^{\text{in}}$  and  $V_{pj}^{\text{out}} = S_{jk} V_{pk}^{\text{in}}$ . The elements of  $S$  can be

computed from the characteristic impedances ( $Z_k$ 's) of the conductors. In computing  $S_{kk}$  and  $S_{jk}$ , all conductors except the conductor  $k$  will be replaced by a load  $Z_{Lk}$  which is the parallel combination of the impedances of the conductors being replaced.

$$S_{kk} = \frac{Z_{Lk} - Z_k}{Z_{Lk} + Z_k} \quad (7)$$

is then simply the reflection coefficient and

$$S_{jk} = \frac{2Z_{Lk}}{Z_{Lk} + Z_k} \quad (8)$$

is the transmission coefficient at the load  $Z_{Lk}$ .

Observe that the signals  $V_p^{\text{in}}$  are coming to the node p from the nodes adjacent to p. Thus these signals are originating at some previous times. Starting from the initial conditions at  $t = 0$ , a given network is simulated at discrete times. At any given time, the signals at all nodes are computed. Therefore, the already computed and stored signals  $V_p^{\text{in}}$  are used to compute the signals  $V_p^{\text{out}}$  from equations

(5). The signals  $V_p^{\text{out}}$  are going out from node p at the present time and these same signals will be treated as signals coming in to the adjacent nodes at appropriate future times. Also observe that, at any given time, the nodes in a circuit can be processed in any order. There will be a scattering